

The Application of the Plane Grating to the Determination of the Index of Refraction of a Gas with Values for Air from $\lambda 2500$ to $\lambda 6500$

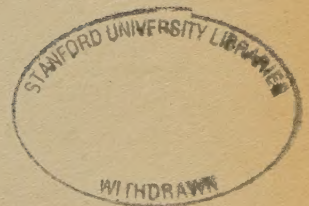
DISSERTATION

SUBMITTED TO THE BOARD OF UNIVERSITY STUDIES OF THE JOHNS HOPKINS
UNIVERSITY IN CONFORMITY WITH THE REQUIREMENTS FOR
THE DEGREE OF DOCTOR OF PHILOSOPHY

BY

ROBERT WILLIAM DICKEY

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THE APPLICATION OF THE PLANE GRATING TO THE DETERMINATION OF THE INDEX OF REFRACTION OF A GAS, WITH VALUES FOR AIR FROM λ 2500 TO λ 6500¹

By ROBERT WILLIAM DICKEY

INTRODUCTION

There are many methods applicable to the determination of the index of refraction of a gas for wave-lengths in the visible spectrum and the near ultra-violet spectrum. Any method, to be of universal use, must give results in the extreme ultra-violet which are accurate to the same degree as those in the visible region. It was the purpose of the present investigation to study the use of the plane grating in extending as far as possible into the ultra-violet region the dispersion curve of a gas. The test of any method is the value of the results derived by it, consequently the index of refraction of air has been obtained for different wave-lengths between 2493 and 6412 Å. A careful study of the possibilities and difficulties of the method has been made.

HISTORICAL

As the problem is an old one, it has been undertaken by a great many investigators. A few of the most important methods will be reviewed along with the results found.

Ketteler,² using a Jamin interferometer, observed the fringe-shift for definite change in pressure of the gas. His difficulty was that of obtaining efficient sources of monochromatic light, although the method gave absolute values. He obtained the index of refraction for yellow sodium light for air, carbon dioxide, hydrogen, sulphur dioxide, and cyanogen.

L. Lorenz,³ working with a similar apparatus, determined the refractive indices of air freed of moisture and carbon dioxide, and

¹ Dissertation, Johns Hopkins University.

² *Poggendorf's Annalen*, 124, 390-406, 1865.

³ *Annalen der Physik und Chemie*, N.F. 11, 70, 1880.

of oxygen and nitrogen for sodium and lithium light. Mascart,¹ making use of Talbot bands, measured the absolute value of the index of refraction of air and chlorine for sodium light and the relative values for the four cadmium lines. Perreau² used the actual tubes of Mascart, but discarded the method involving Talbot bands and obtained a banded spectrum by analyzing white light which had passed through a Jamin refractometer. Kayser and Runge³ placed a hollow prism between a photographic plate and a large concave grating and measured the displacement of various spectral lines when the pressure of the air in the prism was changed. They worked between $\lambda = 5630$ and $\lambda = 2360$ with a pressure-change of 10 atmospheres.

Rentschler⁴ used a Fabry and Perot interferometer and a concave grating and determined the indices of refraction of air, nitrogen, oxygen, carbon dioxide, and carbon monoxide from $\lambda = 5769$ to $\lambda = 3341$. The interferometer plates were silvered, and owing to the low reflecting power of silver between $\lambda = 2280$ and $\lambda = 3341$ Rentschler's results were not carried below $\lambda = 3341$. Among the recent publications of work in the visible spectrum is that of C. and M. Cuthbertson, who used a Jamin refractometer and determined the indices of refraction of air, oxygen, nitrogen, hydrogen,⁵ neon,⁶ krypton, xenon, helium, and argon⁷ between $\lambda = 6563$ and $\lambda = 4861$. Quite recently an investigation of this problem has been carried out by Miss Howell,⁸ using a Fabry and Perot interferometer (with plates nickered by cathode discharge), in connection with a quartz spectrograph. She has extended the dispersion-curve of air to $\lambda = 2652$ and those of hydrogen, oxygen, and carbon dioxide to $\lambda = 2753$. The advantage of this method is that it is not necessary to count the number of interference bands going by for a change of pressure, thus making the method applicable to the ultra-violet region.

¹ *Comptes rendus*, **86**, 321, 1878.

² *Annales de Chimie et de Physique*, **7**, 289, 1896.

³ *Annalen der Physik*, **50**, 293, 1893.

⁴ *Astrophysical Journal*, **28**, 345, 1908.

⁵ *Proc. Roy. Soc.*, **83**, 151, 1909.

⁶ *Ibid.*, **83**, 149, 1909.

⁷ *Ibid.*, **81**, 440, 1908.

⁸ *Physical Review*, **6**, 81, 1915.

The method here described consists in determining the index of refraction of a gas by finding the ratio of the wave-lengths in vacuum and in the gas by means of a Rowland plane grating, using a Littrow type of mounting. The general idea involved is not new, for Osborne and Lester¹ used this method for the determination of the index of refraction of a liquid. Its application to a gas is new and there are many details in which this method differs from the one mentioned above.

THEORETICAL CONSIDERATIONS

If v_0 is the velocity of light-waves in the pure ether, v , the velocity in a definite gas, then the index of refraction of the gas with reference to the ether is defined as the ratio of v_0 to v . Calling the index of refraction n , we have

$$n = \frac{v_0}{v} = \frac{\nu\lambda_0}{\nu\lambda}$$

where ν is the frequency corresponding to the wave-length λ .

$$\therefore n = \frac{\lambda_0}{\lambda} \text{ and } n - 1 = \frac{\lambda_0 - \lambda}{\lambda} = \frac{\Delta\lambda}{\lambda}. \quad (1)$$

Since λ is known, it is necessary to measure $\Delta\lambda$ for a change in pressure of the gas. To accomplish this, the fundamental principle of the grating is made use of. The condition under which we obtain a bright line of the m th order, when white light is allowed to fall upon the grating, is that the path difference between rays reflected from adjacent grooves shall be $m\lambda$, where λ is the wave-length of the line in question. The equation for the Littrow mounting is

$$m\lambda = a(\sin \theta + \sin i) \quad (2)$$

where θ is the angle of incidence of the light, and this is kept constant, and i is the angle of the diffracted beam. i differs very slightly from θ . If the grating is surrounded by a gas, let the path difference for some particular line in the spectrum be $m\lambda_1$. Then when the same grating is placed in vacuum, the path difference between adjacent grooves for the same line will be $m\lambda_2$, according to equation (1), where λ_2 is greater than λ_1 . Since θ is constant in

¹ *Ibid.*, 35, 210, 1912.

equation (2), the change in λ is accompanied by a change in i and a shift of the line is observed. If a photographic plate is placed in the focal plane of the observing telescope, this shift can be recorded by making two exposures, one with the grating in vacuum and the second with the grating in the gas, keeping the grating fixed during the two exposures.

APPARATUS AND METHOD

The Littrow form of spectrometer (Fig. 1) was used. To avoid instrumental shifts due to the possible relative motion of the parts of the apparatus, it was mounted as rigidly as possible. Two 3-inch iron I-beams, 300 cm in length, were fastened together and parallel to each other so that the flanges gave a firm horizontal surface in which to mount the various parts. At one end a cast-iron base-plate 22 cm in diameter and 3.2 cm thick was mounted and insulated from the I-beams by hard rubber supports so as to prevent as much as possible the transference of heat to or from the plate. At the center of this plate the grating-holder was attached with the proper adjustments. Just behind the grating-holder the connection to the pump and outside air was made as shown in Fig. 1. At the other end of the beams an iron plate, carrying the slit, reflecting prism, and photographic plate-holder, was fastened. The collimator lens was mounted on a slide fastened between the I-beams. In focusing for any definite region of the spectrum the collimator lens alone was moved. A heavy brass bell-jar, 12.5 cm inside diameter, 14 cm deep, with walls 1 cm thick and a heavy flange at the bottom, covered the grating. The front was arranged so that a quartz window could be inserted and made air-tight. The opening for the window was 2.5×4 cm. Through a hole in the top of the bell-jar a thermometer reading to tenths of a degree was inserted.

A Rowland 2.5-inch grating, ruled with 15,000 lines to the inch, was used. A grating with bright third and fourth orders was selected. On account of the high coefficient of expansion of speculum metal, every precaution was taken to keep the temperature of the grating constant during a series of exposures. A heating coil wound on the inside walls of a large wooden box ($40 \times 55 \times 75$ cm),

surrounding the bell-jar, helped to keep the temperature constant. Then the massiveness of the bell-jar tended to oppose any rapid changes in the temperature of the inside. This problem of temperature has proved a very serious trouble, and it will be discussed more fully in a later paragraph.

The entire optical system, consisting of a small condensing lens *C*, a totally reflecting prism *P*, the collimator lens *L*, and the plane parallel plate *W*, was made of quartz. As a source of light the

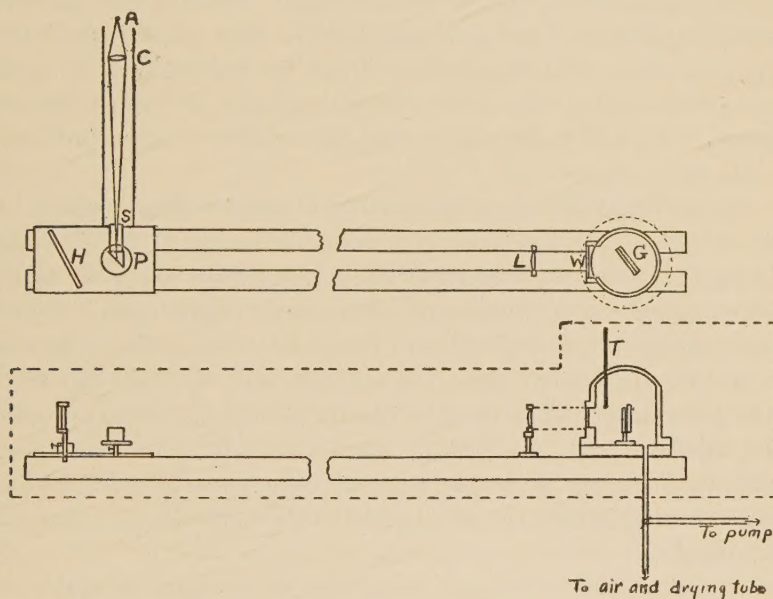


FIG. 1.—Arrangement of apparatus

Pfund iron arc, operated on 110 volts with 5 or 6 amperes, was used. An image of the arc was thrown on the slit *S* by means of the condensing lens *C*, and a larger image was projected on the wall of the room at a distance of 5 meters so as to make certain that the same part of the arc was being used in a series of exposures. The observations were recorded on photographic plates.

The general procedure was to pump out the bell-jar until the pressure was a small fraction of a millimeter of mercury, take an exposure on the photographic plate, then let in the air through

drying solutions and make a second exposure on the same plate, being very careful not to disturb it between exposures. The greatest difficulty to be overcome was the change in temperature of the grating between exposures, due to a thermodynamic increase or decrease in the temperature of the surrounding air when the pressure was increased or decreased. The resultant change in the temperature of the air itself is negligible so far as the density of the air is affected, but the high coefficient of expansion of the metal of the grating causes change in the grating space. This in turn causes a considerable shift of the spectrum lines for even a small fraction of a degree change in temperature. Upon the reduction of the pressure in the bell-jar by about one atmosphere, a sudden drop of about 1° C. was indicated by the thermometer whose bulb was inside the inclosure.

If one could tell when the grating reached a steady state, the difficulty would be overcome, but it is evident that there is a considerable lag of temperature in the grating with reference to the thermometer. A thermo-couple with one set of junctions insulated and imbedded in a brass block attached to the grating was used in an attempt to determine the steady state, but this had to be abandoned owing to the heat conducted to or away from the junction by the leads. An attempt was then made to avoid the difficulty by lowering the temperature of the air to that of liquid air before letting it into the bell-jar, but this method failed and was discarded.

Although this error due to temperature changes could not apparently be done away with, it was decreased to a certain extent by the filling of all the unnecessary space in the bell-jar with pieces of metal of high specific heats and large absorbing surfaces.

Finally, the scheme of lowering and raising the pressure of the air at intervals of twenty minutes and taking all exposures three minutes before and three minutes after letting the air into the bell-jar was adopted. In this way the error due to temperature-changes will be the same for all observations. The foregoing cycle of operations was repeated several times before any observations were made. As the total change in the shift due to thermodynamic changes was only a few thousandths of a millimeter, it was impossible to determine the actual value to a reasonable degree of accu-

racy by taking exposures at different times before and after letting the air into the bell-jar. The only course left open was to take the mean value of the index of refraction of air for some line in the visible spectrum and refer all observations to this one value. The exposures for different regions of the spectrum varied from 15 seconds to 5 minutes, so it was necessary to adopt the three-minute intervals mentioned above.

The spectrum of the third order was used between $\lambda=6400$ and $\lambda=4300$, of the fourth order from $\lambda=4100$ to $\lambda=2700$, and of the third order for the extreme ultra-violet observations. Even though the overlapping spectra were not in focus, much trouble was experienced by the fog produced on the plates. By the proper selection of absorbing solutions and screens this was avoided to a certain extent.

In the design of the apparatus any mechanism for setting the grating without removing the bell-jar was purposely avoided, for it was thought that such an arrangement might cause an instrumental shift in the grating when the pressure in the inclosure was changed. The grating was set at the desired angle by means of a telescope and scale, the grating being used as a mirror, after the sides of the constant temperature box and the bell-jar had been removed. When the bell-jar was replaced over the grating it was necessary to make the plane-parallel quartz window perpendicular to the incident light from the lens. This was done by adjusting the bell-jar until the image of the part of the incident light reflected from the front surface of the quartz plate fell in the center of the photographic plate and just below the spectrum. The location of this image plays an important part in one of the corrections which will be discussed later.

The air was freed from moisture by passing it through sulphuric acid and over phosphorous pentoxide. The pressure in the bell-jar was measured by a mercury manometer, the readings being reduced to mercury at 0° C. Readings of temperature were made to 0.05° C. and of pressure to one-tenth of a millimeter.

RESULTS

Observations were made at intervals of about 200 Å, three or four plates being made for each region. This gave a means of

checking the measurements taken from a single plate. In every case tried the values from two different plates were found to agree.

The position of the lines on the plates and their corresponding shifts were measured by a traveling microscope, the screw of which was made in the same way as the screws for the Rowland ruling machines. Measurements were made to $1/1000$ mm. In cases where the lines on any one plate were numerous, only those best adapted to measurement were selected. Each line and its corresponding shift were measured on an average of six or seven times and the mean of the results taken.

Position correction.—In the preliminary experiments it was noticed that the shift was not only horizontal, but vertical to a small degree. In looking for an explanation of this, it was found that a correction, depending upon the distance of the line from the



FIG. 2

center of the plate, had to be made. This correction can be deduced with the aid of Fig. 2. The only essential parts to be considered are the quartz plate W , which serves as a boundary between the outside air and the inclosure, and the photographic plate. If, when the pressure in space 2 is the same as that in space 1, the path of the ray from the grating G to the photographic plate is ABC , its path, when the pressure in 2 is less than that in 1, will be ABD owing to the deviation of the ray as it passes from a rarer to a denser medium. At the center of the optical system this will not occur, for the ray is normal to the quartz plate W .

Let n' = the index of refraction of space 1 with reference to space 2 at the observed temperature and pressure. This value of n' is derived from measurements at the center of the photographic plate where no correction is necessary.

Let l = distance ED , or distance of line from center of plate

Let θ = tilt of photographic plate

Then $CD = \left(\frac{n'l \cos \theta}{EB} - \frac{l \cos \theta}{EB} \right) EB$, approximately, $= (n' - 1) l \cos \theta$

On account of this error the measured shifts taken from the half of the plate toward the red end of the spectrum are too small, and those of the other half too large. In referring the shifts to the center of the plate, the foregoing correction had to be added to or subtracted from the measured values of the "red" end or "blue" end respectively of the plate. Obviously it was very necessary to determine the center of the photographic plate, and this was done by means of the image of the slit reflected from the front surface of the quartz window. This has been discussed in the section on the apparatus and method.

The values of the shifts, after the foregoing correction had been made, were reduced to standard conditions. As a result of many experiments the following relation between the index of refraction of a substance and its density,

$$\frac{n-1}{\rho} = \text{constant},$$

has been well established by many investigators. Further, the shift for any definite wave-length is such that

$$\frac{n-1}{\Delta\lambda} = \text{constant}.$$

Hence, putting

$$\Delta\lambda = s,$$

it follows that

$$\frac{s}{\rho} = \text{constant}.$$

Or, expressing ρ in terms of pressure and absolute temperature,

$$\frac{sT}{p} = \text{constant}.$$

Hence, if s_1 is observed at temperature T_1 and pressure p_1 (expressed in millimeters of mercury), the value of s , which would have been observed if the temperature had been 0°C . and the pressure 760 mm, is given by the formula

$$\frac{s_{760}}{760} = \frac{s_1(273+t_1)}{p_1}$$

or

$$s = s_1 \frac{760}{p_1} \frac{273 + t_1}{273}.$$

As the shifts reduced to standard conditions were given in millimeters, it was then necessary to reduce them to angstrom units. To do so it was necessary to compute the dispersion for each line of the plate. The positions of three lines, one at the center of the plate, x_0 , and one near each end of the plate, x_1 and x_2 respectively, were carefully measured. It was assumed that any two wave-lengths with their corresponding positions obeyed the following relation,

$$\lambda_0 - \lambda = A(x_0 - x) + B(x_0 - x)^2,$$

in which A and B are constants, x_0 the position of λ_0 on the plate, and x the position of any other line λ . λ_0 is taken as near as possible to the center of the plate. To obtain the values of A and B the three lines mentioned above were used. Differentiating the expression, we have

$$\frac{d\lambda}{dx} = A + 2B(x_0 - x).$$

Then if x_0 is selected, the dispersion at any other point on the plate indicated by x is known from the above relations. All the shifts were then multiplied by the value of $\frac{d\lambda}{dx}$ for the corresponding value of x . The resultant values are the respective changes in the wave-lengths of the different lines of the spectrum as the pressure is changed by the amount observed. According to equation (1), $n - 1 = \frac{\Delta\lambda}{\lambda}$; hence the values of $n - 1$ could be calculated. The wave-lengths of the lines were taken from Kayser's *Handbuch*.

A specimen of the results is given in Table I. This is one of the seventeen tables obtained. In this table column I gives the relative positions of the lines on the photographic plate; column II, the corresponding wave-lengths as taken from Kayser's *Handbuch*; column III, the mean values of the shifts measured; column IV, the shifts corrected for the distance of the line from the center of

TABLE I

I π (mm)	II λ (Å)	III S_s (mm)	IV S_t (mm)	V S (mm)	VI $\frac{d\lambda}{dx} \frac{(\text{Å})}{\text{mm}}$	VII $\Delta\lambda(\text{Å})$	VIII $n-1$	IX
$\lambda_{256.188}$	4199.25	0.5501	0.5338	0.5943	2.1285	1.2650	0.0003012	For $\lambda = 4219.52$ $n-1 = 0.0003011$
258.483	4204.15	.5506	.5349	.5950	2.1270	1.2671	.0003014	
259.964	4206.87	.5509	.5355	.5962	2.1256	1.2673	.0003012	
263.039	4213.82	.5505	.5360	.5968	2.1232	1.2671	.0003007	
264.243	4210.33	.5508	.5366	.5984	2.1223	1.2699	.0003012	
265.740	4219.52	.5514	.5376	.5986	2.1211	1.2696	.0003009	
277.985	4245.43	.5540	.5433	.6049	2.1115	1.2772	.0003009	
288.707	4268.93	.5565	.5486	.6108	2.1032	1.2847	.0003009	
290.574	4271.95	.5562	.5488	.6111	2.1017	1.2843	.0003006	
295.643	4282.59	.5575	.5514	.6139	2.0978	1.2879	.0003007	
297.103	4285.60	.5567	.5510	.6135	2.0966	1.2863	.0003001	For $\lambda = 4315.29$ $n-1 = 0.0003005$
301.239	4294.32	.5587	.5541	.6170	2.0934	1.2916	.0003008	
306.647	4305.03	.5590	.5558	.6188	2.0892	1.2928	.0003003	
307.818	4308.09	.5598	.5569	.6201	2.0883	1.2950	.0003006	
$\lambda_{311.262}$	4315.29	.5605	.5585	.6219	2.0856	1.2970	.0003006	
319.000	Center of plate							
321.803	4337.24	.5624	.5631	.6270	2.0774	1.3025	.0003003	
329.352	4352.90	.5637	.5664	.6306	2.0715	1.3063	.0003001	
337.578	4360.95	.5661	.5709	.6357	2.0651	1.3128	.0003004	
340.558	4376.11	.5662	.5718	.6367	2.0628	1.3134	.0003001	
344.242	4383.71	.5668	.5734	.6384	2.0600	1.3151	.0003000	For $\lambda = 4408.58$ $n-1 = 0.0003001$
354.535	4404.95	.5700	.5792	.6449	2.0519	1.3232	.0003004	
356.323	4408.58	.5697	.5704	.6451	2.0505	1.3238	.0003001	
363.212	4422.74	.5713	.5828	.6489	2.0451	1.3271	.0003001	
365.530	4427.50	.5726	.5847	.6510	2.0433	1.3302	.0003004	
367.145	4430.79	.5713	.5838	.6500	2.0420	1.3273	.0002996	
$\lambda_{372.878}$	4442.52	0.5730	0.5876	0.6543	2.0376	1.3332	0.0003001	

$t_1 = 24.0^\circ \text{C.}$; $\phi_1 = 744.1 \text{ mm.}$; angle of tilt of camera $= 12^\circ$; third order of grating.

the plate; column V, the shifts reduced to standard conditions; column VI, the dispersion in angstrom units per millimeter as measured on the photographic plate; column VII, the shifts in angstrom units, or rather the change in wave-lengths due to the change in pressure; and column VIII the values of $n-1$ as calculated by equation (1). The calculation of the index of refraction from one line alone is not very reliable, for the physical characteristics of the line may be such as to influence the results. For this reason the practice of determining the final values from a number of lines has been adhered to. This is shown in column IX. The values of $n-1$ were reduced to one particular line by taking as nearly as possible the same number of evenly spaced lines on each side of a central line, and taking the mean of the values. This practice is permissible in the region of the spectrum where the dispersion is comparatively small, but in the ultra-violet the separate values of $n-1$ were reduced to one particular line by the use of Kayser and Runge's dispersion formula, and the mean of these reduced values was taken as a final value. Table I represents one of the better sets of observations. In some cases values of $n-1$ had to be determined from a less number of lines.

Since the temperature correction for the shifts was not determined, it was necessary to select some iron line whose index of refraction for air has been well determined by other observers, and refer all values to this line. For this purpose the line $\lambda=4315.29$ was selected. The index of refraction was calculated from the dispersion formulae of Kayser and Runge,¹ Rentschler,² and Miss Howell,³ the values obtained for $n-1$ being 0.0002961, 0.0002959, and 0.0002958. The mean of these is 0.0002959. The value obtained in the present investigation was 0.0003005, thus giving a difference of 0.0000046, which will be called the temperature correction. From the following discussion it will be seen that this correction is constant for all regions of the spectrum; and the corrected values of the index of refraction for air for all wave-lengths is found by subtracting this correction from the computed values.

¹ *Annalen der Physik*, 50, 293, 1893.

² *Astrophysical Journal*, 28, 345, 1908.

³ *Physical Review*, 6, 81, 1915.

The formula for the Littrow type of mounting of the plane grating is

$$m\lambda = a(\sin \theta + \sin i)$$

where m is the order of the spectrum, λ the wave-length of the spectrum line, a the grating space, θ the angle the incident light makes with the normal to the grating, and i the angle of the refracted beam. For the present discussion θ and i can be considered as remaining constant. Suppose a temperature-change in the grating occurs, then the expansion produces a change, δa , in the grating space. The corresponding change in the wave-length, $\delta\lambda$, is given by the relation

$$m\delta\lambda = \delta a(\sin \theta + \sin i).$$

Dividing this expression by the one above, we have

$$\frac{\delta\lambda}{\lambda} = \frac{\delta a}{a} = \text{constant},$$

if the change δa is a constant in every case. The exposures were carried out under exactly the same conditions so that δa would be a constant. Therefore the correction to $n-1$ for temperature changes is a constant for all values of λ .

The indices of refraction of air for the different wave-lengths obtained in this manner are given in Table II. The first column contains the values of the wave-lengths, the second, the observed values of $n-1$, and the third the corrected values of $n-1$. In the fourth column are recorded the number of lines observed in determining $n-1$ for the specified wave-length, λ .

The dispersion-curve is shown in Fig. 3. Curve I indicates the values obtained from the observed values of the shifts of the lines on the photographic plate, while curve II represents the values of $n-1$ referred to the known value for the line $\lambda=4315.29$.

It is customary to express dispersion-curves in terms of the Cauchy formula,

$$n-1 = a + \frac{b}{\lambda^2} + \frac{c}{\lambda^4},$$

although this is of no theoretical importance. It is given here so that the results can be compared with those of other observers. According to Cuthbertson (*loc. cit.*)

$$n-1 = 10^{-7} \left(2885.4 + \frac{13.38}{\lambda^2} + \frac{.50}{\lambda^4} \right);$$

Kayser and Runge (*loc. cit.*) find

$$n-1 = 10^{-7} \left(2878.7 + \frac{13.16}{\lambda^2} + \frac{.316}{\lambda^4} \right);$$

Rentschler (*loc. cit.*) finds

$$n-1 = 10^{-7} \left(2903 + \frac{3.80}{\lambda^2} + \frac{1.23}{\lambda^4} \right);$$

Miss Howell (*loc. cit.*) finds

$$n-1 = 10^{-7} \left(2881.7 + \frac{11.83}{\lambda^2} + \frac{.46}{\lambda^4} \right)$$

where λ is expressed in thousandths of a millimeter. In the present investigation the values of the constants a , b , and c in the Cauchy formula were obtained by the method of least mean squares, and the following formula was found:

$$n-1 = 10^{-7} \left(2878.12 + \frac{11.59}{\lambda^2} + \frac{0.551}{\lambda^4} \right).$$

This is in good agreement with the formula of Kayser and Runge and that of Miss Howell.

DISCUSSION OF RESULTS

A remarkably smooth dispersion-curve was obtained, only one point failing to touch the curve. This was at $\lambda = 4879.39$, and the reason for this can be found in the physical characteristics of the lines in this region. As they were broad and not well defined, accurate settings on them were almost impossible. For the converse reason the values below $\lambda = 4500$ are more accurate.

The magnitude of the shift is also a determining factor in the accuracy of the results as well as the sharpness of the lines. To increase the magnitude of the shift, higher orders of the grating were used as far as possible, in keeping with the intensity of the

TABLE II

λ (Å)	$n-1$ (Observed)	$n-1$ (Corrected)	No. Lines Used in Determining $n-1$
6411.90.....	0.0002956	0.0002910	3
6230.93.....	.0002960	.0002914	4
5615.89.....	.0002965	.0002919	8
5406.02.....	.0002968	.0002922	8
5328.21.....	.0002971	.0002925	4
5269.70.....	.0002971	.0002925	3
5202.49.....	.0002972	.0002926	9
5139.64.....	.0002975	.0002929	1
4879.39.....	.0002986	.0002940	7
4710.46.....	.0002988	.0002942	2
4618.95.....	.0002991	.0002945	7
4517.70.....	.0002997	.0002951	9
4408.58.....	.0003001	.0002955	7
4315.29.....	.0003005	.0002959	12
4219.52.....	.0003011	.0002965	7
4118.70.....	.0003015	.0002969	18
3941.03.....	.0003023	.0002977	12
3867.35.....	.0003027	.0002981	10
3651.61.....	.0003043	.0002997	12
3575.60.....	.0003049	.0003003	3
3450.46.....	.0003058	.0003012	12
3292.73.....	.0003076	.0003030	11
3037.51.....	.0003107	.0003061	9
2727.64.....	.0003165	.0003119	14
2664.77.....	.0003184	.0003138	4
2613.91.....	.0003205	.0003159	11
2567.01.....	.0003228	.0003182	8
2493.34.....	0.0003268	0.0003222	4

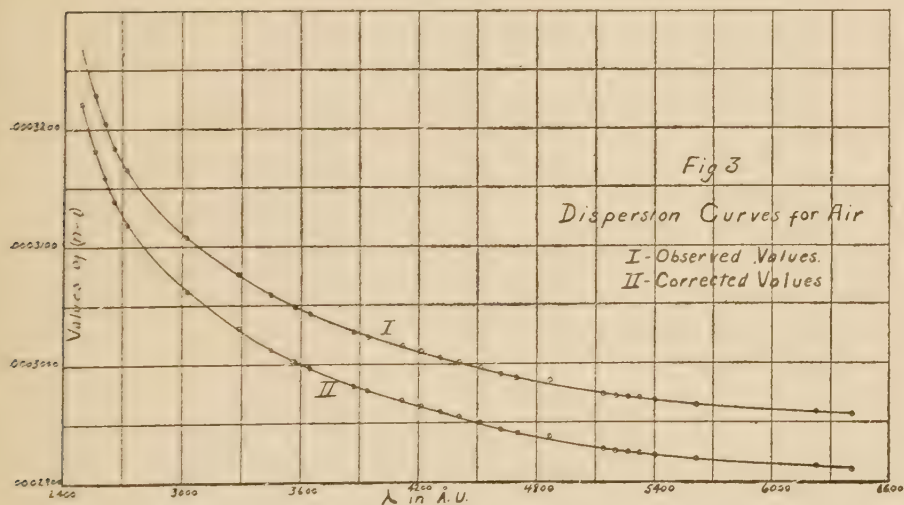


FIG. 3

lines. The large angle of tilt of the photographic plate in the extreme ultra-violet made it possible to use the third order in this region and obtain the same magnitude of shift. The shifts varied from 0.45 mm to 0.92 mm, the particular value depending upon the order used and the region of the spectrum.

The dispersion-curve was not extended below $\lambda = 2493$, owing to the arrangement of the apparatus. The opening in the bell-jar was such as greatly to reduce the amount of the grating space used and consequently to cut down the intensity of the spectral lines. From a consideration of the number of separate measurements which are associated with a single value of $n - 1$ and the good agreement of these measurements, it is safe to say that the fourth significant figure is determined to a fair degree of accuracy. It can be relied upon to be correct to within several units.

As far as the method is concerned, there seems to be only one serious difficulty—the effect of temperature-changes as the pressure of the gas is changed. It is believed that, with several improvements in the present apparatus, the value of this temperature correction can be determined and consequently the absolute value of $n - 1$. The author hopes to continue this work in the near future, using other gases. In the present form this method is applicable to gases which do not injure the grating and for which a single value of the index of refraction is known.

Among the advantages of this method the following may be mentioned: first, any source of light may be used which gives fairly sharp lines, the number being immaterial provided there are enough in any one region to determine the dispersion; second, the final value of $n - 1$ is not obtained from observations on a single line; third, values of $n - 1$ can be obtained for wave-lengths of all lines that can be photographed in a reasonable length of time using an ordinary grating spectrometer. This last fact makes the method a very desirable one in extending into the extreme ultra-violet the dispersion-curves of gases.

SUMMARY

1. The application of the plane grating to the determination of the index of refraction of gases has been thoroughly studied and

the only serious difficulty is found to be one due to changes of temperature.

2. The dispersion-curve of air has been obtained from

$$\lambda = 6412 \text{ to } \lambda = 2493.$$

3. The accuracy of the corrected values of the index of refraction of air agrees with those obtained by other methods.

In conclusion I wish to thank Dr. Anderson, who suggested the present problem and whose interest, help, and constant encouragement have been invaluable.

I am indebted to Professor Ames for his interest and advice throughout the course of the work. I wish also to thank Dr. Pfund for his advice and helpful suggestions.

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BIOGRAPHICAL SKETCH

Robert William Dickey, son of Robert J. and Mattie (Jones) Dickey, was born May 13, 1891, in Bath County, Virginia, and was prepared for college in the high school of Covington, Virginia. In 1906 he entered Washington and Lee University, receiving the degrees of B.S., B.A., and M.A. in the years 1910, 1911, and 1912, respectively. While there he was assistant in physics for five years, besides holding the physics and civil engineering scholarships, the Bradford Scholarship, and the Howard Houston Fellowship for two years. During the year 1912-13 he was instructor in physics and mathematics in Washington and Lee University. In 1913 he entered the Johns Hopkins University as a graduate student in physics, and has been a holder of a Virginia Scholarship (1913-1914), assistant in physics (1914-1915), and Fellow in physics (1915-1916). In 1915 he was admitted to the degree of Master of Arts. He has pursued courses of instruction under Professors Ames, Wood, Whitehead, Anderson, and Pfund in physics, applied electricity, and astronomy, and under Dr. Cohen in mathematics.